

Mössbauer neutrinos

Joachim Kopp

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in collaboration with E. Kh. Akhmedov and M. Lindner
based on [arXiv:0802.2513](#) (JHEP **0805** (2008) 005),
[arXiv:0803.1424](#) (J. Phys. **G36** (2009) 078001),
[arXiv:0904.4346](#) (JHEP **0906** (2009) 049)

Outline

- 1 The Mössbauer neutrino experiment
- 2 Oscillations of Mössbauer neutrinos: Qualitative arguments
- 3 Mössbauer neutrinos in QFT
 - The formalism
 - Inhomogeneous line broadening
 - Homogeneous line broadening
 - Natural line broadening
- 4 Can the same results be obtained in a QM approach?
- 5 Conclusions

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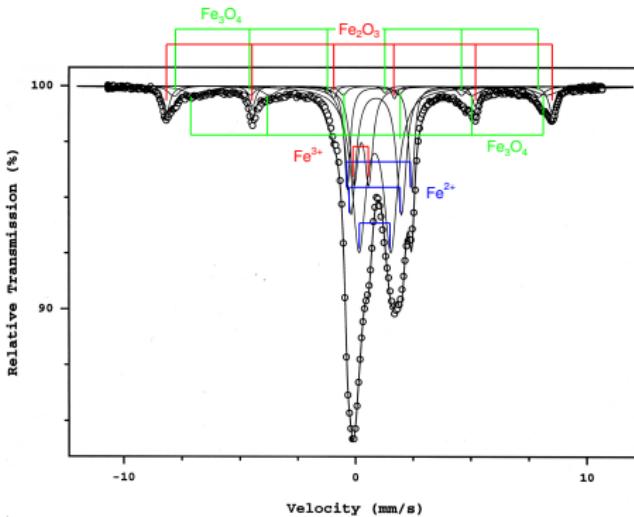
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A similar effect should exist for neutrino emission/absorption in bound state β decay and induced electron capture processes.

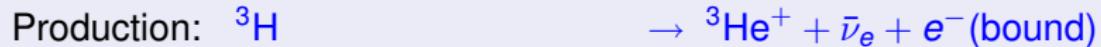
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Proposed experiment:



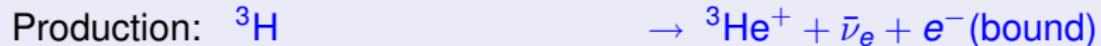
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Physics goals:

- Neutrino oscillations on a laboratory scale: $E = 18.6 \text{ keV}$, $L_{\text{atm}}^{\text{osc}} \sim 20 \text{ m.}$
- Gravitational interactions of neutrinos
- Study of solid state effects with unprecedented precision

Mössbauer neutrinos (2)

Mössbauer neutrinos have very special properties:

- Neutrino receives *full* decay energy: $Q = 18.6 \text{ keV}$
- Natural line width: $\gamma \sim 1.17 \times 10^{-24} \text{ eV}$
- Actual line width: $\gamma \gtrsim 10^{-11} \text{ eV}$
 - ▶ Inhomogeneous broadening (Impurities, lattice defects)
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Experimental challenges:

- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth $\gamma \gtrsim 10^{-11} \text{ eV}$ be achieved?
- Can the resonance condition be fulfilled?

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Theoretical questions:

- Can Mössbauer neutrinos oscillate? S. M. Bilenky, F. v. Feilitzsch, W. Potzel
- What is the effect of line broadening on oscillations?
- Comparison of different formalisms (QM \leftrightarrow QFT)

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Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} (\bar{e}_{\alpha L} \gamma^\mu U_{\alpha j} \nu_{jL}) W_\mu^- + \text{diag. mass terms} + h.c.$$

(flavour eigenstates: $\alpha = e, \mu, \tau$, mass eigenstates: $j = 1, 2, 3$)

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Assume, at time $t = 0$ and location $\vec{x} = 0$, a flavour eigenstate

$$|\nu(0, 0)\rangle = |\nu_\alpha\rangle = \sum_i U_{\alpha j}^* |\nu_j\rangle$$

is produced. At time t and position \vec{x} , it has evolved into

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Oscillation probability:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \langle \nu_\beta | \nu(t, \vec{x}) \rangle \right|^2 = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i(E_j - E_k)t + i(\vec{p}_j - \vec{p}_k)\vec{x}}$$

Equal energies or equal momenta?

Typical *assumptions* in the “textbook derivation” of the oscillation formula:

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These are *assumptions* or *approximations*, not fundamental principles!

Equal energies or equal momenta? (2)

- In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.

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Energy-momentum conservation for emission of mass eigenstate $|\nu_i\rangle$:

$$E_j^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_j^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^4}{4m_\pi^2}$$

$$p_j^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_j^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^4}{4m_\pi^2}$$

For massless neutrinos: $E_j = p_j = E \equiv \frac{m_\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \text{ MeV}$.

To first order in m_j^2 :

$$E_j \simeq E + \xi \frac{m_j^2}{2E}, \quad p_j \simeq E - (1 - \xi) \frac{m_j^2}{2E}, \quad \xi \approx \frac{1}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Oscillation probability for Mössbauer neutrinos

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 - Requires neither equal E nor equal p
 - Takes into account finite resolutions of the source and the detector

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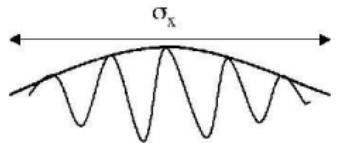
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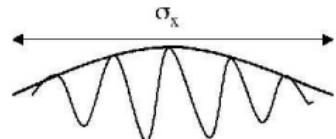
Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...

Conditions for oscillations in a wave packet approach



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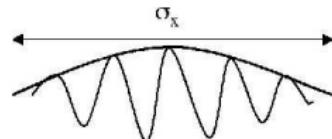


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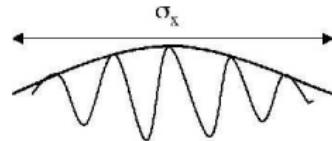


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$$\sigma_m^2 = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2} > \Delta m^2$$

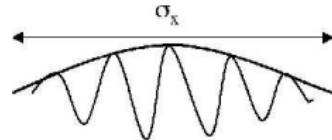
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This is easily fulfilled for Mössbauer neutrinos, since

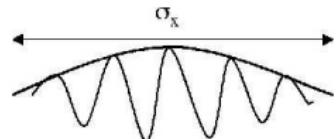
$$\sigma_E \sim 10^{-11} \text{ eV}$$

$$\sigma_p = 1/2\sigma_x \sim 1/\text{interatomic distance} \sim 10 \text{ keV}$$

$$E = p = 18.6 \text{ keV}$$

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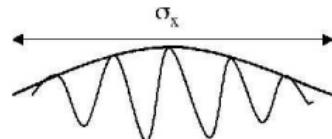
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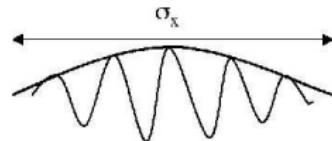
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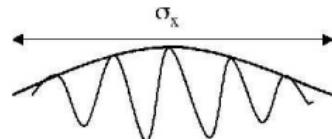
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It can be shown that, for Mössbauer neutrinos, σ_p is small enough, so that

$$L^{\text{osc}} \ll L^{\text{coh}}.$$

⇒ Standard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[-2\pi i \frac{L}{L_{jk}^{\text{osc}}} \right]$$

$$L_{jk}^{\text{osc}} = \frac{4\pi E}{\Delta m_{jk}^2}$$

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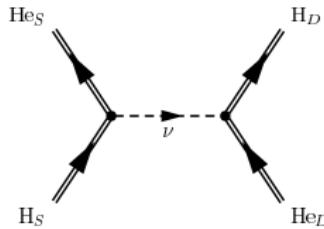
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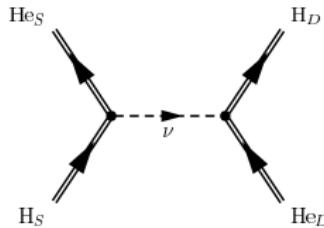
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External particles reside in harmonic oscillator potentials.
E.g. for ${}^3\text{H}$ atoms in the source:

$$\psi_{\text{H},s}(\vec{x}, t) = \left[\frac{m_{\text{H}}\omega_{\text{H},s}}{\pi} \right]^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},s} |\vec{x} - \vec{x}_s|^2 \right] \cdot e^{-iE_{\text{H},s}t}$$

Oscillation amplitude

$$i\mathcal{A} = \int d^3x_1 dt_1 \int d^3x_2 dt_2 \left(\frac{m_{\text{H}}\omega_{\text{H},S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{\text{H},S}t_1}$$
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$$\cdot \left(\frac{m_{\text{He}}\omega_{\text{He},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{He}}\omega_{\text{He},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{-iE_{\text{He},D}t_2}$$
$$\cdot \left(\frac{m_{\text{H}}\omega_{\text{H},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{+iE_{\text{H},D}t_2}$$
$$\cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)}$$
$$\cdot \bar{u}_{e,S} \gamma_\mu (1 - \gamma^5) \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} (1 + \gamma^5) \gamma_\nu u_{e,D}.$$

Oscillation amplitude

$$i\mathcal{A} = \int d^3x_1 dt_1 \int d^3x_2 dt_2 \left(\frac{m_H \omega_{H,S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_H \omega_{H,S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{H,S}t_1}$$
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Evaluation:

- $dt_1 dt_2$ -integrals → energy-conserving δ functions → p_0 -integral trivial
- $d^3x_1 d^3x_2$ -integrals are Gaussian
- d^3p -integral: Use **Grimus-Stockinger theorem** (for large $L = |\vec{x}_D - \vec{x}_S|$).

W. Grimus, P. Stockinger, Phys. Rev. D54 (1996) 3414, hep-ph/9603430

The Grimus-Stockinger theorem

Let $\psi(\vec{p})$ be a three times continuously differentiable function on \mathbb{R}^3 , such that ψ itself and all its first and second derivatives decrease at least like $1/|\vec{p}|^2$ for $|\vec{p}| \rightarrow \infty$. Then, for any real number $A > 0$,

$$\int d^3 p \frac{\psi(\vec{p}) e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \rightarrow \infty} -\frac{2\pi^2}{L} \psi(\sqrt{A} \frac{\vec{L}}{L}) e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}).$$

⇒ Quantification of requirement of on-shellness for large $L = |\vec{L}|$.

From the amplitude to the transition rate

Amplitude:

$$iA = \frac{-i}{2L} \mathcal{N} \delta(E_S - E_D) \exp \left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2} \right] \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 e^{i\sqrt{E_S^2 - m_j^2} L} \\ \cdot \bar{u}_{e,S} \gamma_\mu \frac{1-\gamma^5}{2} (\not{p}_j + m_j) \frac{1+\gamma^5}{2} \gamma_\nu u_{e,D},$$
$$\sigma_p^{-2} = (m_H \omega_{H,S} + m_{He} \omega_{He,S})^{-1} + (m_H \omega_{H,D} + m_{He} \omega_{He,D})^{-1}$$

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Transition rate: Integrate $|\mathcal{A}|^2$ over densities of initial and final states

$$\Gamma \propto \int_0^\infty dE_{H,S} dE_{He,S} dE_{He,D} dE_{H,D} \\ \cdot \delta(E_S - E_D) \rho_{H,S}(E_{H,S}) \rho_{He,D}(E_{He,D}) \rho_{He,S}(E_{He,S}) \rho_{H,D}(E_{H,D}) \\ \cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \underbrace{\exp \left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right]}_{\text{Analogue of Lamb-Mössbauer factor}} \underbrace{e^{i(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2})L}}_{\text{Oscillation phase}}$$

The Lamb-Mössbauer factor

The **Lamb-Mössbauer factor** is the relative probability of recoil-free emission and absorption, compared to the total emission and absorption probability.

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Convenient reformulation:

$$\exp \left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right] = \exp \left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2} \right] \exp \left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2} \right]$$

where $(p_{jk}^{\min})^2 = E_S^2 - \max(m_j^2, m_k^2)$.

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where $(p_{jk}^{\min})^2 = E_S^2 - \max(m_j^2, m_k^2)$.

⇒ **Localization condition**

$$4\pi\sigma_x E / \sigma_p \lesssim L_{jk}^{\text{osc}},$$

(with $\sigma_x = 1/2\sigma_p$) is satisfied if $L_{jk}^{\text{osc}} \gtrsim 2\pi\sigma_x$, which is easily fulfilled in realistic situations.

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Inhomogeneous line broadening

Energy levels of ${}^3\text{H}$ and ${}^3\text{He}$ in the source and detector are smeared e.g. due to crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079

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$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

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$$\Gamma \propto \exp \left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2} \right] \exp \left[-\frac{|\Delta m^2|}{2\sigma_p^2} \right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}} \\ \cdot \left\{ 1 - 2s^2c^2 \left[1 - \frac{1}{2}(e^{-L/L_S^{\text{coh}}} + e^{-L/L_D^{\text{coh}}}) \cos \left(\pi \frac{L}{L_{\text{osc}}} \right) \right] \right\}$$

$$L_{S,D}^{\text{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

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$$L_{S,D}^{\text{coh}} = 4\bar{E}^2/\Delta m^2 \gamma_{S,D}$$

In realistic cases: $L_{S,D}^{\text{coh}} \gg L^{\text{osc}} \Rightarrow$ Decoherence is not an issue.

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- Fluctuating electromagnetic fields in solid state crystal
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- Ansatz: Introduce modulation factors of the form

$$f_{A,B}(t) = \exp \left[-i \int_0^t dt' [E_{A,B}(t') - E_{A,B,0}] t' \right]$$

in the ${}^3\text{H}$ and ${}^3\text{He}$ wave functions ($A = \text{H}, \text{He}$, $B = S, D$).

J. Odeurs, Phys. Rev. **B52** (1995) 6166

Transition amplitude for homogeneous line broadening

$$\begin{aligned} i\mathcal{A} = & \int d^3x_1 dt_1 \int d^3x_2 dt_2 \left(\frac{m_H \omega_{H,S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_H \omega_{H,S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{H,S}t_1} \\ & \cdot \left(\frac{m_{He} \omega_{He,S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{He} \omega_{He,S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{+iE_{He,S}t_1} \\ & \cdot \left(\frac{m_{He} \omega_{He,D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{He} \omega_{He,D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{-iE_{He,D}t_2} \\ & \cdot \left(\frac{m_H \omega_{H,D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_H \omega_{H,D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{+iE_{H,D}t_2} \\ & \cdot \sum_j \mathcal{M}_S^\mu \mathcal{M}_D^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} \exp [-ip_0(t_2 - t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)] \\ & \cdot \bar{u}_{e,S} \gamma_\mu (1 - \gamma^5) \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} (1 + \gamma^5) \gamma_\nu u_{e,D} \end{aligned}$$

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Evaluation:

- $d^3x_1 d^3x_2$ -integrals are Gaussian
- d^3p -integral: Use **Grimus-Stockinger theorem**.

Transition rate for homogeneous line broadening

Transition rate $\Gamma \propto \langle A A^* \rangle$

(statistical average of $A A^*$ over all possible ${}^3\text{H}$ and ${}^3\text{He}$ states).

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Transition rate $\Gamma \propto \langle \mathcal{A} \mathcal{A}^* \rangle$

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⇒ We encounter the quantity

$$\begin{aligned} B_S(t_1, \tilde{t}_1) &\equiv \left\langle f_{\text{H},S}(t_1) f_{\text{He},S}^*(t_1) f_{\text{H},S}^*(\tilde{t}_1) f_{\text{He},S}(\tilde{t}_1) \right\rangle \\ &= \left\langle \exp \left[-i \int_{\tilde{t}_1}^{t_1} dt' \Delta E_S(t') \right] \right\rangle, \end{aligned}$$

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- Markov approximation: $\langle \Delta E_S(t') \Delta E_S(t'') \rangle = \gamma_S \delta(t' - t'')$

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$$\Rightarrow B_S(t_1, \tilde{t}_1) = \exp \left[-\frac{1}{2} \gamma_S |t_1 - \tilde{t}_1| \right].$$

Transition rate for homogeneous line broadening (2)

Result:

$$\Gamma \propto \exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}} \\ \cdot \left\{ 1 - 2s^2c^2 \left[1 - \frac{1}{2}(e^{-L/L_S^{\text{coh}}} + e^{-L/L_D^{\text{coh}}}) \cos\left(\pi \frac{L}{L^{\text{osc}}}\right) \right] \right\}$$

... identical to the result for inhomogeneous line broadening.

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Amplitude for broadening by natural line width

Take into account the instability of ${}^3\text{H}$ in the source and the detector.

$$\begin{aligned} i\mathcal{A} = & \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\text{H}}\omega_{\text{H},S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{\text{H},S}t_1} \\ & \cdot \left(\frac{m_{\text{He}}\omega_{\text{He},S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{He}}\omega_{\text{He},S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{+iE_{\text{He},S}t_1} \\ & \cdot \left(\frac{m_{\text{He}}\omega_{\text{He},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{He}}\omega_{\text{He},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{-iE_{\text{He},D}t_2} \\ & \cdot \left(\frac{m_{\text{H}}\omega_{\text{H},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{+iE_{\text{H},D}t_2} \\ & \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\ & \cdot \bar{u}_{e,S} \gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2} \gamma_\nu u_{e,D} \end{aligned}$$

(correctness of this formula can be verified in the Wigner-Weisskopf approach)

Amplitude for broadening by natural line width

Take into account the instability of ${}^3\text{H}$ in the source and the detector.

$$\begin{aligned} i\mathcal{A} = & \int d^3x_1 \int_0^T dt_1 \int d^3x_2 \int_0^T dt_2 \left(\frac{m_{\text{H}}\omega_{\text{H},S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{\text{H},S}t_1 - \frac{1}{2}\gamma t_1} \\ & \cdot \left(\frac{m_{\text{He}}\omega_{\text{He},S}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{He}}\omega_{\text{He},S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{+iE_{\text{He},S}t_1} \\ & \cdot \left(\frac{m_{\text{He}}\omega_{\text{He},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{He}}\omega_{\text{He},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{-iE_{\text{He},D}t_2} \\ & \cdot \left(\frac{m_{\text{H}}\omega_{\text{H},D}}{\pi} \right)^{\frac{3}{4}} \exp \left[-\frac{1}{2} m_{\text{H}}\omega_{\text{H},D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{+iE_{\text{H},D}t_2 - \frac{1}{2}\gamma(T-t_2)} \\ & \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \\ & \cdot \bar{u}_{e,S} \gamma_\mu \frac{1-\gamma^5}{2} \frac{i(\not{p} + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \frac{1+\gamma^5}{2} \gamma_\nu u_{e,D} \end{aligned}$$

(correctness of this formula can be verified in the Wigner-Weisskopf approach)

E. Akhmedov, J. Kopp, M. Lindner, JHEP 0805 (2008) 005 (arXiv:0802.2513)

Probability for broadening by natural line width

$$\begin{aligned} \mathcal{P} \propto & \sum_{j,k} \theta(T_{jk}) |U_{ej}|^2 |U_{ek}|^2 \\ & \cdot \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right] e^{i(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2})L} \\ & \cdot e^{-\gamma T_{jk}} e^{-L/L_{jk}^{\text{coh}}} \frac{\sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j})\right] \sin\left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k})\right]}{(E_S - E_D)^2} \end{aligned}$$

where $T_{jk} = \min\left(T - \frac{L}{v_j}, T - \frac{L}{v_k}\right)$ and $L_{jk}^{\text{coh}} = \frac{4\bar{E}^2}{\gamma |\Delta m_{jk}^2|}$

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- Oscillation term: $e^{i(\sqrt{\bar{E}^2 - m_j^2} - \sqrt{\bar{E}^2 - m_k^2})L}$
- Lamb-Mössbauer factor: $\exp\left[-(p_{jk}^{\min})^2/\sigma_p^2\right]$
- Localization term: $\exp\left[-|\Delta m_{jk}^2|/2\sigma_p^2\right]$
- Coherence term: $e^{-L/L_{jk}^{\text{coh}}}$

Probability for broadening by natural line width (2)

- Resonance term

$$\frac{\sin \left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_j}) \right] \sin \left[\frac{1}{2}(E_S - E_D)(T - \frac{L}{v_k}) \right]}{(E_S - E_D)^2}$$

does *not* depend on γ , but *only* on the total measurement time T (Heisenberg principle).

Probability for broadening by natural line width (2)

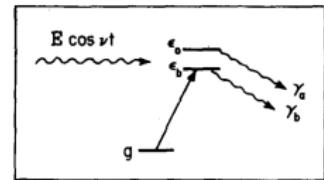
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- Analogy: **subnatural spectroscopy** in quantum optics

- ▶ Atom is excited instantaneously to state $|b\rangle$.
- ▶ Continuous irradiation with frequency ν .
- ▶ Probability for exciting state $|a\rangle$ is proportional to $[(\nu - \nu_{\text{res}})^2 + (\gamma_a - \gamma_b)^2/4]^{-1}$, not $[(\nu - \nu_{\text{res}})^2 + (\gamma_a + \gamma_b)^2/4]^{-1}$.



P. Meystre, O. Scully, H. Walther, Optics Communications 33 (1980) 153

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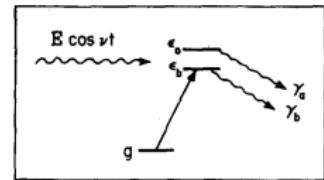
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- Here:

- ▶ $|b\rangle \Leftrightarrow {}^3\text{H}$ atom in the source, ${}^3\text{He}$ atom in the detector
- ▶ $|a\rangle \Leftrightarrow {}^3\text{He}$ atom in the source, ${}^3\text{H}$ atom in the detector
- ▶ Excitation of $|b\rangle \Leftrightarrow$ Production of source
- ▶ Transition $|b\rangle \rightarrow |a\rangle \Leftrightarrow$ neutrino production, propagation and absorption

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Lorentzian wave packets

Describe Mössbauer neutrino as a Lorentzian wave packet:

$$\langle p | \bar{\nu}_{es}(t) \rangle = \frac{1}{N_s} \sum_j U_{ej} f_{js} \frac{\sqrt{\gamma_s/2\pi}}{p - p_{js} + i\gamma_s/2} \exp [-iE_j t] |\nu_j\rangle$$

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Detection process: Projection onto

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Fudge factors

$$f_{jS} \equiv \exp \left[\frac{\bar{E}^2 - m_j^2}{2\sigma_{pS}^2} \right], \quad f_{jD} \equiv \exp \left[\frac{\bar{E}^2 - m_j^2}{2\sigma_{pD}^2} \right]$$

describe dependence of production/detection amplitudes on neutrino mass
(cannot be computed in QM, but only in QFT).

Transition amplitude, probability, and rate in QM

Transition amplitude:

$$\mathcal{A}(t, L) = \int dp \langle \bar{\nu}_{eD} | p \rangle \langle p | \bar{\nu}_{eS}(t) \rangle$$

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$$\mathcal{P}(L) = \frac{1}{T} \int_{-T/2}^{T/2} dt \mathcal{A}^*(t, L) \mathcal{A}(t, L)$$

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Transition rate:

$$\Gamma = \frac{1}{4\pi L^2} \Gamma_0^{\text{MB}} \mathcal{P}(L) \sigma^{\text{MB}}$$

Transition rate in QM

Approximations:

- Use that momentum space wave packets are very narrow
- Use that neutrinos are ultra-relativistic ($m_j \ll \bar{E}_j$)

Result:

$$\Gamma \propto \exp\left[-\frac{E_{S,0}^2 - m_2^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m^2|}{2\sigma_p^2}\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}} \\ \cdot \left\{ 1 - 2s^2c^2 \left[1 - \frac{1}{2}(e^{-L/L_S^{\text{coh}}} + e^{-L/L_D^{\text{coh}}}) \cos\left(\pi \frac{L}{L^{\text{osc}}}\right) \right] \right\}$$

... identical to QFT result.

Comparison of QFT and QM approaches

QFT

QM

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QFT

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Few input parameters:

$$\omega_{H,He;S,D}, E_{H,He;S,D}, \gamma_{S,D}$$

Many input parameters:

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Shape of neutrino wave packets
determined automatically

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Comparison of QFT and QM approaches

QFT	QM
Few input parameters: $\omega_{H,He;S,D}$, $E_{H,He;S,D}$, $\gamma_{S,D}$	Many input parameters: Γ_0^{MB} , σ^{MB} , $p_{jS,D}$, $\gamma_{S,D}$, $\xi_{S,D}$, $f_{jS,D}$,
Shape of neutrino wave packets determined automatically	Shape of neutrino wave packets put in by hand
Predicts total transition rate (in- cluding Lamb-Mössbauer factor)	Predicts only oscillation proba- bility

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Correct localization condition

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No (or incomplete) localization
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Comparison of QFT and QM approaches

QFT	QM
Few input parameters: $\omega_{H,He;S,D}$, $E_{H,He;S,D}$, $\gamma_{S,D}$	Many input parameters: Γ_0^{MB} , σ^{MB} , $p_{JS,D}$, $\gamma_{S,D}$, $\xi_{S,D}$, $f_{JS,D}$
Shape of neutrino wave packets determined automatically	Shape of neutrino wave packets put in by hand
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Realistic implementation of line broadening	Line broadening parameterized by $\gamma_{S,D}$

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Abstract formalism	More transparent formalism

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Conclusions

- Mössbauer neutrinos do oscillate.
- Plane wave treatment: Mössbauer neutrinos are the *only* case where the equal energy assumption is justified.
- Wave packet considerations:
 - ▶ Coherence and localization conditions are irrelevant for realistic experiments.
 - ▶ Properties of the neutrino wave packets have to be put in by hand.
- QFT treatment:
 - ▶ Only properties of the source and the detector are put in by hand.
 - ▶ Generalized Lamb-Mössbauer factor leads to localization condition.
 - ▶ Nonzero line width (due to homogeneous and inhomogeneous line broadening) leads to coherence condition.
 - ▶ Both conditions are easily fulfilled in realistic experiments.
 - ▶ Natural line width dominance unrealistic, but interesting analogy to subnatural spectroscopy → Resolution *not* limited by line width!
- QM treatment:
 - ▶ Lorentzian wave packets
 - ▶ QFT result can be reproduced
 - ▶ ... but only if many parameters are adjusted by hand
 - ▶ QM is the less abstract, but also the less complete formalism

Thank you!

Oscillation formula for neutrino wave packets

Assume Gaussian wave packets:

$$|\nu_\alpha(x, t)\rangle = \frac{1}{(2\pi\sigma_{ps}^2)^{1/4}} \sum_j U_{\alpha j}^* \int \frac{dp}{\sqrt{2\pi}} e^{-(p-p_{js})^2/4\sigma_{ps}^2} e^{-i\sqrt{p^2+m_j^2}t+ipx} |\nu_j\rangle$$

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Oscillation formula:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[-2\pi i \frac{L_{jk}^{\text{osc}}}{L_{jk}^{\text{coh}}} - \left(\frac{L_{jk}^{\text{osc}}}{L_{jk}^{\text{coh}}} \right)^2 - 2\pi^2 \xi^2 \left(\frac{1}{2\sigma_p L_{jk}^{\text{osc}}} \right)^2 \right]$$

C. Giunti, C. W. Kim, U. W. Lee, Phys. Rev. **D44** (1991) 3635; C. Giunti, C. W. Kim, Phys. Lett. **B274** (1992) 87
K. Kiers, S. Nussinov, N. Weiss, Phys. Rev. **D53** (1996) 537, hep-ph/9506271

C. Giunti, C. W. Kim, Phys. Rev. **D58** (1998) 017301, hep-ph/9711363, C. Giunti, Found. Phys. Lett. **17** (2004) 103, hep-ph/0302026

with

$L_{jk}^{\text{osc}} = 4\pi E / \Delta m_{jk}^2$	Oscillation length
$L_{jk}^{\text{coh}} = 2\sqrt{2}E^2 / \sigma_p \Delta m_{jk}^2 $	Coherence length
E	Energy that a massless neutrino would have
ξ	quantifies the deviation from E (tiny for Mössbauer neutrinos)
σ_p	Effective wave packet width (tiny for Mössbauer neutrinos)

Results from the wave packet treatment

- Decoherence term

$$\exp \left[-\frac{L}{L_{jk}^{\text{coh}}} \right]$$

cannot inhibit oscillations because

$$L_{jk}^{\text{coh}} / L_{jk}^{\text{osc}} \sim E / \sigma_p \sim 10^{15}.$$

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⇒ Our expectation is confirmed:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i \frac{\Delta m_{jk}^2 L}{2E}}.$$

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We are looking for a formalism, in which these quantities are automatically determined from the properties of the source and the detector.

Probability for broadening by natural line width (3)

- Time dependence for $E_S = E_D$: $T^2 e^{-\gamma T}/4$.

Classical argument for this behaviour:

- ▶ Numbers of ${}^3\text{H}$ nuclei in the detector, N_D , and in the source, N_S , obey

$$\dot{N}_D = -\dot{N}_S N_0 P_{ee} \frac{\sigma(T)}{4\pi L^2} - \gamma N_D,$$

where N_0 is the number of ${}^3\text{He}$ atoms in the detector.

- ▶ Absorption cross section $\sigma(T) \simeq s_0 T$, because Heisenberg principle requires resonance condition to be fulfilled to an accuracy $\sim T^{-1}$. For Lorentzian emission an absorption lines, the overlap integral is then proportional to T .
- ▶ Using $N_S = N_{S,0} \exp(-\gamma T)$, the solution to the above equation is

$$N_D = \frac{N_{S,0} N_0 \gamma P_{ee} s_0}{8\pi L^2} T^2 e^{-\gamma T}.$$

The time-energy uncertainty relation

Mandelstam-Tamm relation:

$$\Delta E \Delta O \geq \frac{1}{2} \left| \frac{d}{dt} \overline{O}(t) \right|.$$

Here, $\overline{O}(t) = \langle \psi(t) | O | \psi(t) \rangle$, for any operator O and QFT Fock state $|\psi(t)\rangle$.

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Choose $O \equiv |\nu_\alpha\rangle\langle\nu_\alpha|$ (projection in 3d flavour space) and $\psi(t)$ a neutrino state

$$\Delta E \geq \frac{1}{2} \frac{\left| \frac{d}{dt} P(t) \right|}{\sqrt{P(t) - P^2(t)}},$$

with $P(t) = |\langle \nu_\alpha | \psi(t) \rangle|^2$.

S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. **G35** (2008) 095003 (arXiv:0803.0527)

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S. M. Bilenky, F. v. Feilitzsch, W. Potzel, J. Phys. G35 (2008) 095003 (arXiv:0803.0527)

Problem: Interpretation of $P(t)$.

- Would be the oscillation probability **for a completely delocalized detector**.
- Imagine wave packet which is large compared to L_{osc} : Delocalized detector averages out oscillations.
- More realistic: **Projection in flavour space and in coordinate space**:

$$O_{\vec{x}} \equiv |\nu_\alpha\rangle |\vec{x}\rangle \langle \vec{x}| \langle \nu_\alpha|$$

The time-energy uncertainty relation (2)

Mandelstam-Tamm relation now reads:

$$\Delta E \geq \frac{1}{2} \frac{|\frac{d}{dt} P(\vec{x}, t)|}{\sqrt{P(\vec{x}, t) - P^2(\vec{x}, t)}},$$

with $P(\vec{x}, t) = |\langle \vec{x} | \langle \nu_\beta | \Psi(t) \rangle|^2$ (now indeed an oscillation probability).

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Two-flavour approximation: $P(\vec{x}, t) = 1 - \sin^2 2\theta \sin^2 \phi(\vec{x}, t)$

$$\Rightarrow \Delta E \geq |E_1 - E_2| \frac{\sin 2\theta \cos \phi(\vec{x}, t)}{\sqrt{1 - \sin^2 2\theta \sin^2 \phi(\vec{x}, t)}}.$$

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Certainly fulfilled, if fulfilled for $\sin^2 2\theta = 1$.

$$\Rightarrow \Delta E \geq |E_1 - E_2|.$$

“Energy difference of mass eigenstates must be smaller than energy uncertainty.”

Easily fulfilled for Mössbauer neutrinos due to large *momentum* uncertainty.